An Improved Light Beam Search Method for Multiobjective Optimizations of Inverse Problems

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An improved light beam search method is proposed to solve multiobjcetive designs of inverse problems. To guarantee a uniform distribution of the searched Pareto solutions, the utopia plane (line) is divided into sub-domains and an evaluation mechanism is proposed to identify the topology relationship between each individual and the preference zone. To avoid pre-mature and to enhance the diversity of the population, a multi-external achieves methodology is proposed, with the goal of using different achieves in the mating pool evolutionary process in different searching stages of the algorithm. Mathematical test functions and a benchmark inverse problem, TEAM Workshop Problem 22, are used to testify the effectiveness and the efficiency of the proposed method. The results demonstrate that the proposed method can obtain well distributed Pareto solutions under the predefined directions of the decision maker's preference with a less iteration number.

***Index Terms*—Decision making, heuristic algorithms, inverse problems, Pareto optimization.**

I. An Improved Light Beam Search Method

A. Light Beam Search Method

FROM a practical engineering perspective, a whole Pareto front of a multiobjective design problem is not always necessary. Instead, only a special part is attractive to a decision maker (DM). To take a DM's preference into consideration in finding the Pareto front of a multiobjective design, a light beam search (LBS) method based on evolutionary multiobjective optimizations is proposed in [1]. Different from those based on reference points [2] and desirability functions [3], LBS method uses a direction defined by an aspiration point and a reservation point as the searching direction and a veto threshold to determine the desirable zone of Pareto front. However, in the original LBS method the desirable number of Pareto solutions is determined by both the veto threshold and the distance parameter ε, which will increase the complexity of the algorithm; and also a lot of iterations are needed to obtain the preference Pareto solutions. To simplify the algorithm and speed up the convergence, some improvements are proposed.

B. Sub-domain Division and Evaluation

In order to simplify the algorithm, the middle point is set as the crossing point of the "light direction" and the utopia plane (line) in the proposed method. Moreover, the veto threshold is defined as the neighbor zone on the utopia plane (line) around the middle point. In this way, the middle point and the veto threshold are known at the beginning of the searching process, and there is no need to evolve and calculate the middle point as well as the corresponding neighbor veto threshold through the whole searching phase. Sub-domains are defined by dividing the utopia plane (line) within the preference zone using a predefined step. An even step distribution can lead to a uniform sampling of the Pareto front. Sub-domain center is the geometry center of one sub-domain, which is used to compute the projection distance. A projection distance is the Euclidean distance between the sub-domain center and the projection point of an individual, which is used to criticize a specified individual on the same sub-domain. The probability of an individual projected on the same sub-domain to be selected as a candidate for next cycle of iterations is inversely proportional to its projection distance.

To intuitively demonstrate the sub-domain division, a two objective minimizing problem is used. As shown in Fig. 1, Pareto solutions are those that any improvement in one objective can only occur through the worsening at least one other objective. *z_r* and *z_v* are the aspiration point and the reservation point supplied by the DM, which determine the searching direction. *z_c* is the middle point obtained by the searching direction and the utopia plane (line). *A*; *B* are the anchor points that correspond to the best possible values for respective individual objectives. Line *AB* is the utopia line determined by the anchor points. v_1 and v_2 are the veto thresholds. Zone *CD* is a preference zone on line *AB*, divided into some evenly distributed sub-domains using a predefined step; d_1 and d_2 are the projection distance of individuals E_1 and E_2, respectively. Individual E_2 is better than E_1 both in the rank and the projection evaluation, meanings that it has a high priority being selected in the mating pool selection process.

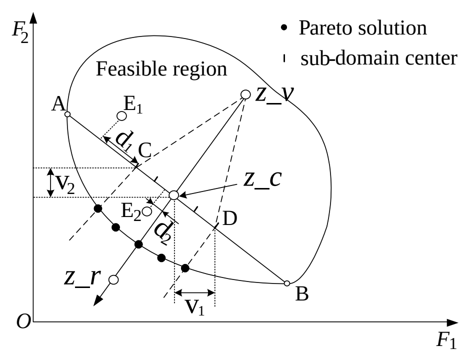

Fig. 1. Intuitive demonstration of sub-domain division

C. Multi-External Achieves

To enhance the diversity of the population and also provide different options to the mating pool, a multi-external archive structure is used in the proposed method. More specially, A_1 is defined as the achieve of the Pareto solutions with smallest projection distances in each sub-domain; A_2 is used to keep the so far best solutions having both highest Pareto rank and smallest projection distance; A_3 is consist of the whole Pareto solutions. At the beginning of the searching procedures, the size of A_1 is small and the main purpose of the search is to explore enough objective space and converge to the Pareto front; consequently, the mating pool evolves based on A_1, A_2 and A_3. However, as the searching process continues, the main goal of the algorithm is transferred to locate the Pareto solutions precisely; the mating pool is thus chosen based on the competition of A_1 and A_2.

D. Algorithm Description

To facilitate the understanding of the proposed LBS method, its iterative procedures are summarized as follows:

Step 1 Define the aspiration point *z_r*; the reservation point *z_v*; the veto threshold *v*; calculate the utopia plane (line) and the middle point *z_c*.

Step 2 Divide the utopia plane (line) into sub-domains.

Step 3 Define the multi-external archives A_1, A_2, A_3. Define *N* is as the maximum iteration number. Initialize the iterative number *t*=0; Generate the initial population using Latin hypercube sampling [4].

Step 4 If t> *N*, go to Step 6. Otherwise, calculate the function values and the projection value of the individuals. Classify the individuals into the sub-domains and calculate the projection distance. Update the A_1, A_2, A_3 based on both Pareto rank using the fast non-dominated sorting approach and the projection distance.

Step 5 Mating pool is selected based on the duality tournament contest mechanism from A_1, A_2, A_3. Generate new population; Go to Step 4.

Step 6 Stop the algorithm.

II. NUMERICAL RESULTS

To validate and demonstrate the advantages of the proposed algorithm, two test functions (MOP2, MOP4) [5] and the TEAM Workshop problem22 [6] are solved.

The parameters of the proposed algorithm for solving MOP2 are set as: *nd* =5, *z_r*=[0 0], *z_ v*=[1 1], *v*=[0.05 0.05], *N*=200. Fig. 2 gives the searched Pareto front using the proposed method, which demenstrates that the proposed method can find a preference segment of the whole Pareto front. The parameters of the proposed algorithm for solving MOP4 are set as: *nd* =[5 20], *z_r*=[-20 -12; -20 -6], *z_ v*=[-14 -6; -14 0], *v*=[0.1 0.1;0.5 0.5], *N*=200. Fig. 3 presents the searched Pareto solutions using the proposed method, demonstrating that the proposed method can find the desired multi-directions of the Pareto front in a single run. The algorithm parameters for solving the TEAM Workshop Problem 22 are set as: *nd* =10, *z_r*=[0 0], *z_ v*=[0.08 0.08], *v*=[0.009 0.02], *N*=200. Fig. 4 depicts the searched Pareto solutions. The averaged iterative numbers used by the original LBS method and the proposed one are 20000 and 11361 with the same maximum iterative number *N*, respectively.

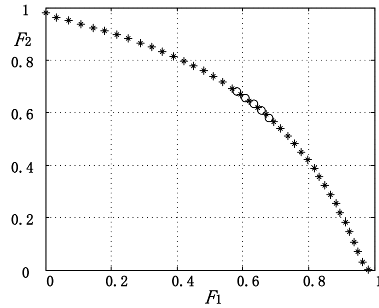

Fig. 2. The searched Pareto front of MOP2: ✱ by using NBI method, ○ by using the proposed method.

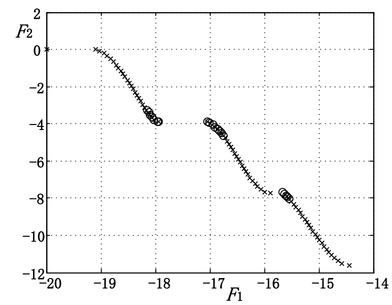

Fig. 3. The searched Pareto front of MOP4: × by using NBI method, ○ by using the proposed method.

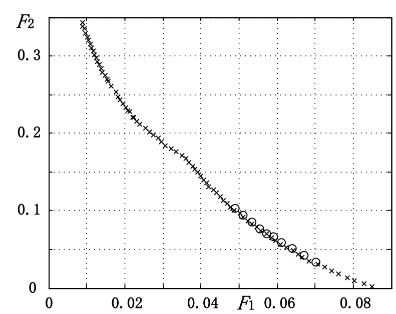

Fig. 4. The searched Pareto front of Problem 22: × by using the NBI method, ○ by using the proposed method.

Obviously, the proposed algorithm can find the arbitrary segments of the complete Pareto front under the preference of a DM with a relative small number of iterations compared to the original LBS method.

III. REFERENCES

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